UNIT - III

3 Fuzzy Decision Making

3.1 : Fuzzy Decision Making

Q.1 What is fuzzy decision making ?

Ans. : Fuzzy decision making is the collection of single or multicriteria techniques aiming at selecting the best alternative in case of imprecise, incomplete, and vague data.

Q.2 List the various types of fuzzy decision making.

Ans. : Various kind of fuzzy decision making are

a. Individual decision making
b. Multi-person decision making
c. Multi-object decision making
d. Multi-attribute decision making
e. Fuzzy Bayesian decision making

Q.3 What are steps for decision making ?

Ans. : The steps involved in the decision-making process are as follows :

1. Determining the set of alternatives : In this step, the alternatives from which the decision has to be taken must be determined.
2. Evaluating alternative : the alternatives must be evaluated so that the decision can be taken about one of the alternatives.
3. Comparison between alternatives : In this step, a comparison between the evaluated alternatives is done

Q.4 Explain following decision making :
a. Individual decision making
b. Multi-person decision making

Ans. : a) Individual decision making :

- Only a single person is responsible for taking decisions.
- A decision situation in this model is characterized by :
  - i. Set of possible actions
  - ii. Set of goals $P_i (i \in x_n)$, expressed in terms of fuzzy set
  - iii. Set of constraints $Q_i (j \in x_m)$, expressed in terms of fuzzy sets.

b) Multi-person decision making :

- When decision are made by many persons, the difference of it from the individual decision maker is :
  1. The goals of single decision makers differ, such that each places a different ordering arrangements.
  2. The individual decision makers have access to different information upon which to base their decision.
- In this case, each member of a group of n single decision makers has a preference ordering P_k, $K \in N_n$, which totally or partially orders a set x.

3.2 : Particle Swarm Optimization

Q.5 What is an agent ? What is a Swarm ? Define swarm intelligence. Give an examples of Swarms in nature.

Ans. : • An agent is anything that can be viewed as perceiving its environment through sensors and acting upon that environment through effectors.

- Swarm is a loosely structured collection of interacting agents. Agents is an individuals that belong to a group. They contribute to and benefit from the group. They can recognize, communicate, and/or interact with each other.

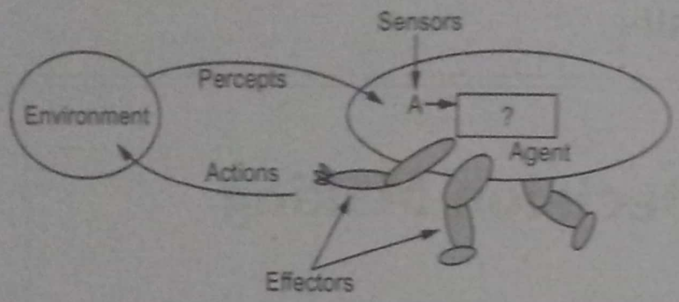


Fig. Q.5.1

- The instinctive perception of swarms is a group of agents in motion, but that does not always have to be the case.
- A swarm is better understood if thought of as agents exhibiting a collective behavior.
- Examples of Swarms in Nature :
  1. Classic Example : Swarm of Bees
  2. Can be extended to other similar systems : Ant colony (Agents : ants), Flock of birds(Agents : birds), Traffic(Agents : cars) , Crowd(Agents : humans), Immune system(Agents : cells and molecules)
- Swarm Intelligence : Any attempt to design algorithms or distributed problem-solving devices inspired by the collective behavior of social insect colonies and other animal societies.
- Swarm intelligence (SI) is an artificial intelligence technique based around the study of collective behavior in decentralized, self-organized systems

Q.6 List the advantages of swarm intelligence.

Ans. : • The systems are scalable because the same control architecture can be applied to a couple of agents or thousands of agents.

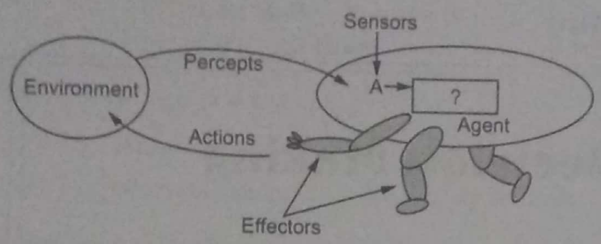
- The systems are flexible because agents can be easily added or removed without influencing the structure.
- The systems are robust because agents are simple in design, the reliance on individual agents is small, and failure of a single agents has little impact on the system's performance.
- The systems are able to adapt to new situations easily.

3.3 : Particle Swarm Optimization (PS Algorithm

Q.7 Write short note on particle sw optimization

Ans. : • Particle Swarm Optimization (PSO) i population based stochastic optimization techn developed by Dr. Eberhart and Dr. Kennedy in 1 inspired by social behavior of bird flocking or f schooling.

- PSO system combines local search methods w global search methods, attempting to bala exploration and exploitation.
- Suppose a group of birds is searching food in area. Only one piece of food is available. Birds not have any knowledge about the location of t food. But they know how far the food is from th present location.
- **So what is the best strategy to locate the food The best strategy is to follow the bird nearest the food.**
- A flying bird has a position and a velocity at a time. In search of food, the bird changes h position by adjusting the velocity. The veloc changes based on his past experience and also t feedbacks received from his neighbor.
- This searching process can be artificially simulate for solving non-linear optimization problem. So th is a population based stochastic optimizatio technique inspired by social behaviour of bir flocking or fish schooling.
- PSO shares many similarities with evolutionar computation techniques such as Genetic Algorithms The system is initialized with a population o random solutions and searches for optima b updating generations.
- In PSO algorithms, the population $P = \{p_1, \dots, p_n\}$ o the feasible solutions is often called a swarm. Th feasible solutions $p_1, \dots, p_n$ are called particles.
- However, unlike GA, PSO has no evolution operators such as crossover and mutation. In PSO the potential solutions, called particles, fly through the problem space by following the current optimum particles.



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- Suppose a group of birds is searching food in an area. Only one piece of food is available. Birds do not have any knowledge about the location of the food. But they know how far the food is from their present location.
- **So what is the best strategy to locate the food ? The best strategy is to follow the bird nearest to the food.**
- A flying bird has a position and a velocity at any time. In search of food, the bird changes his position by adjusting the velocity. The velocity changes based on his past experience and also the feedbacks received from his neighbor.
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- PSO shares many similarities with evolutionary computation techniques such as Genetic Algorithms. The system is initialized with a population of random solutions and searches for optima by updating generations.
- In PSO algorithms, the population $P = \{p_1,\ldots,p_n\}$ of the feasible solutions is often called a **swarm**. The feasible solutions $p_1,\ldots,p_n$ are called **particles.**
- However, unlike GA, PSO has no evolution operators such as crossover and mutation. In PSO, the potential solutions, called particles, fly through the problem space by following the current optimum particles.

- Each particle keeps track of its coordinates in the problem space which are associated with the best solution (fitness) it has achieved so far.
- The fitness value is also stored. This value is called pbest. Another "best" value that is tracked by the particle swarm optimizer is the best value, obtained so far by any particle in the neighbors of the particle. This location is called lbest. When a particle takes all the population as its topological neighbors, the best value is a global best and is called gbest.
- The particle swarm optimization concept consists of, at each time step, changing the velocity of (accelerating) each particle toward its pbest and lbest locations. Acceleration is weighted by a random term, with separate random numbers being generated for acceleration toward pbest and lbest locations.
- In past several years, PSO has been successfully applied in many research and application areas. It is demonstrated that PSO gets better results in a faster, cheaper way compared with other methods.
- **Advantages and disadvantages**
  1. The fitness function can be non-differentiable
  2. There is no general convergence theory applicable to practical, multidimensional problems.

Q.8 Compare Particle Swarm Optimization (PSO) with Genetic Algorithms (GA).

Ans. : • Unlike GAs, PSOs do not change the population from generation to generation, but keep the same population, iteratively updating the positions of the members of the population (i.e., particles).

- PSOs have no operators of "mutation", "recombination", and no notion of the "survival of the fittest".
- On the other hand, similarly to GAs, an important element of PSOs is that the members of the population "interact", or "influence" each other.
- GA usually converges towards a local optimum or even arbitrary points rather than the global optimum of the problem while as PSO tries to find the global optima.
- GA is discrete in nature, i.e. it changes the variables into binary 0's and 1's, and therefore it can easily handle discrete problems, and PSO is continuous and hence must be modified in order to handle discrete problems.

Q.9 Write an algorithm of particle swarm optimization.

Algorithm parameters :

A	:	Population of agents
p_i	:	Position of agent a_i in the solution space
f	:	Objective function
v_i	:	Velocity of agent's a_i
$V(a_i)$	:	Neighborhood of agent a_i (fixed)

- Particle update rule : $p = p + v$

With $v = v + c_1^* \text{ rand } * (\text{pBest} - p) + c_2^* \text{ rand } * (\text{gBest} - p)$

where

p	:	Particle's position
v	:	Path direction
c_1	:	Weight of local information
c_2	:	Weight of global information
pBest	:	Best position of the particle
gBest	:	Best position of the swarm
rand	:	Random variable

Algorithm :

1. Create a 'population' of agents (particles) uniformly distributed over X.
2. Evaluate each particle's position according to the objective function.
3. If a particle's current position is better than its previous best position, update it.
4. Determine the best particle (according to the particle's previous best positions).
5. Update particles' velocities :

$$v_i^{t+1} = \underbrace{v_i^t}_{\text{inertia}} + \underbrace{c_1 U_1^t (pb_i^t - p_i^t)}_{\text{personal influence}} + \underbrace{c_2 U_2^t (gb^t - p_i^t)}_{\text{social inflence}}$$

6. Move particles to their new positions :

$$p_i^{t+1} = p_i^t + v_i^{t+1}$$

7. Go to step 2 until stopping criteria are satisfied.

Q.10 What is Binary Particle Swarm Optimization (BPSO) ? How does it work ?

Ans. : • In BPSO, each solution in the population is a binary string. Each binary string is of dimension "n" which is evaluated to give parameter values.

- In the binary PSO, each binary string represents a particle. Strings are updated bit-by-bit based on its current value, the value of that bit in the best (fitness) of that particle to date, and the best value of that bit to date of its neighbors.
- For binary strings, neighbors can be selected in one of several ways. Some examples are : (for a neighbourhood of size k). Neighbours are the k binary strings whose Hamming distance is minimized. For equal Hamming distances, the choices are arbitrary.
- In BPSO, bit-by-bit updates are done probabilistically. In other words, for a chosen bit "d" in a chosen string "i" it is changed to a 1 with a probability "P" that is a function of its predisposition to be a 1, the best value of itself to date, and the best value of its neighbors.
- The (1 – P) is the probability of changing to a zero.Once P is determined, a random number "R" is generated. If R < P, then the bit becomes a 1; otherwise it becomes a zero.
- In BPSO, population has a set of particles. Each individual particle represents a binary decision.
- This decision can be represented by either YES/TRUE=1 or NO/FALSE=0.
- All particles represent their positions through binary values which are 0 or 1.Velocity is restricted within the range {0,1}
- The velocity vector equation and position vector equation are defined as :

1. Velocity vector equation :

$$v_i^n(t+1) = \frac{1}{1+e^{-v_i^n(t)}}$$

2. Position vector equation :

$$x_i^n\ (t+1) = \begin{cases} 1 & \text{if} \quad r < v_i^n \\ 0 & \text{otherwise} \end{cases}$$

where r is the random number selected from a uniform distribution in [0, 1].

Q.11 Explain flow diagram showing the particle swarm optimization algorithm with Pseudo code.

Ans. : Flow diagram : (See on next page)

Pseudo Code :

```
for each particle
{
   Initialize particle
}
Do until maximum iterations or minimum error criteria
{
   For each particle
   {
      Calculate Data fitness value
      If the fitness value is better than pBest
      {
         Set pBest = current fitness value
      }
      If pBest is better than gBest
      {
         Set gBest = pBest
      }
   }
      For each particle
   {
      Calculate particle Velocity
      Use gBest and Velocity to update particle Data
   }
```

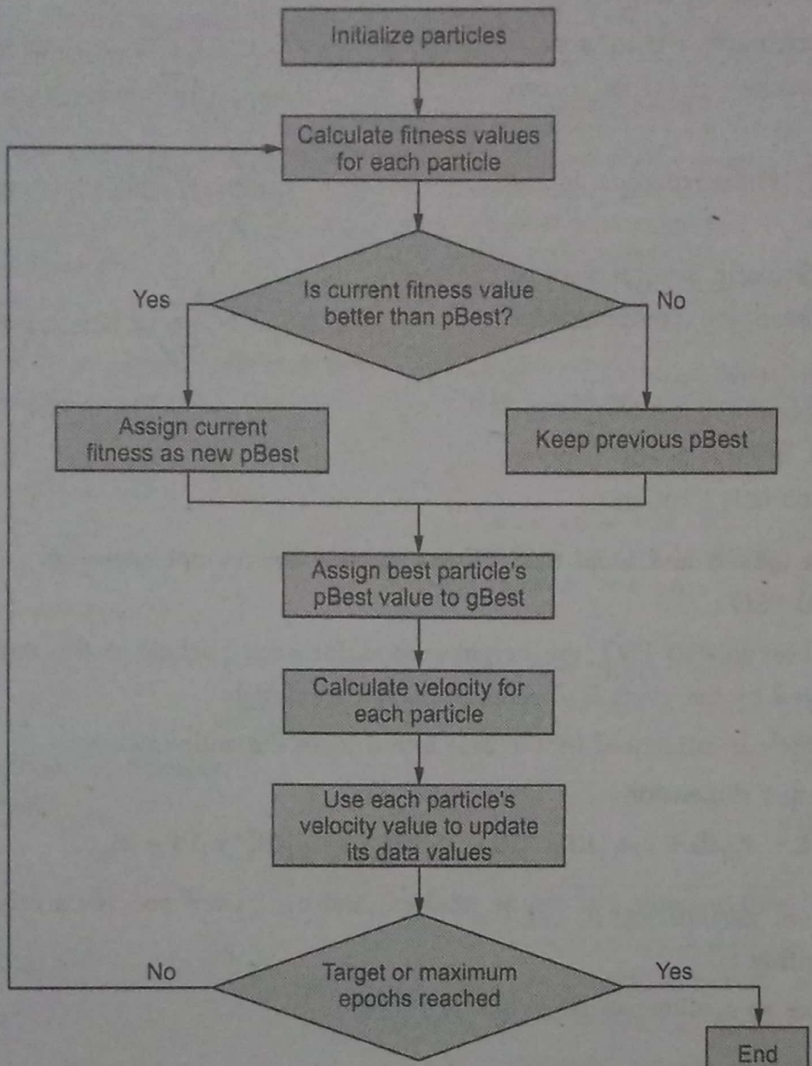

Fig. Q.11.1

Q.12 Discuss topology used in particle swarm optimization.

Ans. : • In PSO, there have been two basic topologies used.

1. Ring Topology (neighborhood of 3)

2. Star Topology (global neighborhood)

Star Topology :

- Fig. Q.8.1 shows star topology.
- gbest model : Each particle is influenced by all the other particles.
- The fastest propagation of information in a population.
- Particles can get stuck easily in local minima.

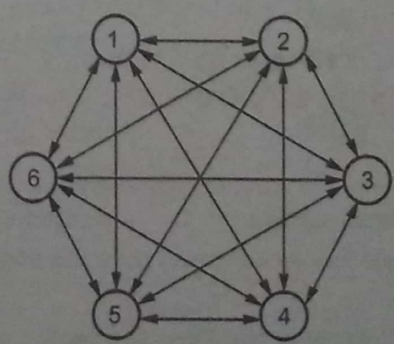

Fig. Q.12.1

- Star neighborhood topology has greater connectivity and interaction between particles compared with other topologies.
- Star neighborhood topology enables fast convergence of the algorithm.

Ring Topology :

- Fig. Q.12.2 shows ring topology.
- lbest model : Each particle is influenced only by particles in its own neighbourhood.
- The propagation of information is the slowest.
- Doesn't get stuck easily in local minima but might increase the computational cost.
- Note that, neighborhood topologies are usually defined by the particle index, and not by the particle location.

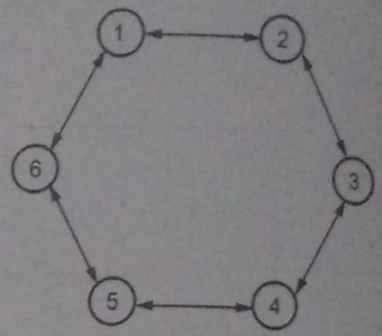

Fig. Q.12.2

Q.13 Explain global best (gbest) and local best (lbest) particle swarm optimization.

Ans. : Global best (gbest) PSO :

- For the global best (gbest) PSO, the neighborhood for each particle is the entire swarm. The social network employed by the gbest PSO reflects the star topology.
- gbest : each particle is influenced by the best found from the entire swarm.
- Velocity update per dimension :

$$v_{ij}(t+1) = v_{ij}(t) + c_1 r_{1j}(t)[y_{ij}(t) - x_{ij}(t)] + c_2 r_{2j}(t)[\hat{y}_j(t) - x_{ij}(t)]$$

$v_{ij}(0) = 0$ (usually, but can be random) and c_1, c_2 are positive acceleration coefficients.

gbest PSO Algorithm :

```
Create and initialize an n_x-dimensional swarm, S;
 repeat
    for each particle i = 1, . . . , S.n_s do
     if f(S·y_i) < f(S·y) then
       S·y_i = S·x_i ;
      end
     if f(S·y_i) < f(S·^y) then
         S·^y = S·y_i ;
    end
  end
for each particle i = 1, . . . , S·n_s do
update the velocity and then the position;
end
until stopping condition is true ;
```

Local best PSO

- The local best (lbest) PSO, uses a ring social network topology where smaller neighborhoods are defined for each particle.
- The social component reflects information exchanged within the neighborhood of the particle reflecting local knowledge of the environment.

- With reference to the velocity equation, the social contribution to particle velocity is proportional to the distance between a particle and the best position found by the neighborhood of particles.
- Due to the larger particle interconnectivity of the gbest PSO, it converges faster than the lbest PSO. However, this faster convergence comes at the cost of less diversity than the lbest PSO.
- The lbest model is less vulnerable to the attraction of local optima but with a slower convergence speed than the gbest model.

lbest PSO algorithm :

```
Create and initialize and n_x-dimensiional swarm;
  repeat
    for each particle i = 1 ... n_s do
      //set the personal best position
      if f(x_i) < f (y_i) then
        y_i = x_i;
      end
      //set the neighborhood best position
      if f (y_i) < f (ŷ_i) then
        ŷ = y_i;
    end
  end
  for each particle i = 1,...n_s do
    update the velocity
    update the position
  end
until stopping condition is true;
```

Q.14 Why used PSO ?

Ans. :

- With a population of candidate solutions, a PSO algorithm can maintain useful information about characteristics of the environment.
- PSO, as characterized by its fast convergence behavior, has an in-built ability to adapt to a changing environment.
- Some early works on PSO have shown that PSO is effective for locating and tracking optima in both static and dynamic environments.

Q.15 Explain difference between gbest and lbest PSO.

gbest PSO	lbest PSO
Global best particle swarm optimization uses a "star" neighborhood topology where all the particles communicate with each other.	Local best particle swarm optimization can be used with different neighborhood topologies with the "ring" topology being the most common.
gbest PSO converges faster compared with local best particle swarm optimization.	lbest PSO converges is slower compared with global best particle swarm optimization.
For the global best PSO, the neighborhood for each particle is the entire swarm.	In lbest PSO, smaller neighborhoods are defined for each particle.
The gbest PSO performs best for unimodal problems.	In lbest, ring structure to provide better performance in terms of the quality of solutions found for multi-modal problems than the star structure.

Fill in the Blanks for Mid Term Exam

Q.1 Swarm is a loosely structured collection of interacting ________.

Q.2 Particle swarm optimization is a ________ based stochastic optimization technique

Q.3 When a particle takes all the population as its topological neighbors, the best value is a global best and is called ________.

Q.4 In BPSO, each solution in the population is a ________ string.

Q.5 The social network employed by the gbest PSO reflects the ________ topology

Q.6 The ________ best PSO, uses a ring social network topology where smaller neighbourhoods are defined for each particle.